Remote Pairs, a Particular Case of an XY chain

Remote Pairs (RP) is an advanced elimination method developed by A. Stuart by the generalization of the Naked/Locked Pair basic procedure. [**1**]

In essence, the **RP** technique consists of "a collection" of Naked Pairs linked together, which posses the property that at the intersection of the marginal cells there can't exist any of the digits which form the cells. [**2**]

In this paper we present a different approach to **RP**, from the general to the particular case: **RP is a very particular case of XY Chain, which we will name XY/RP chain.**

This presentation is addressed to medium level Sudoku players.

FIRST PART

1) XY Chain is a string made up from bivalue cells, which contain only 2 candidates each. The starting cell and the end cell contain the same candidate, for example X. The initial and the final cell are antagonistic cells/ have simultaneously different values/, one can eliminate the X candidates from the intersection of the spheres of influence of these two cells. The logical procedure behind an XY-Chain is shown in fig. 1. [**3**]

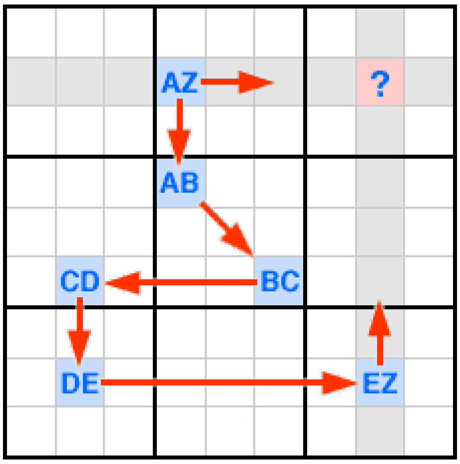

Figure 1

If cell r2c4 = Z, then Z is eliminated from r2c8; if r2c4=A, then r4c4 is B, r6c6 is C, r6c2 is D, r8c2 is E and r8c8 is Z. Z is eliminated from cell r2c8.

In cell r2c8 at the intersection the houses of the initial and final cell, Z will be eliminated, whichever of the placed digits (Z or A) with which the initial cell will be filled. In reference **[4]** is explained the conexion through alternating links of a cell in a XY-chain.

The chain is formed exclusively out of cell which contain 2 candidates; each link in the chain is between 2 common candidates, which belong to a pair of cells from the same house. If a chain of links is formed and the same candidate in the final cells is not linked, one can replace this common candidate at the intersection of the end points.

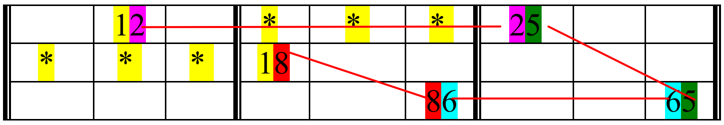

Fig. 2 /pedagogical example/

In Fig. 2 the first link between the cell r1c2 and r1c7 is formed with the common candidate the digit 2; the second link with the candidates of digit 5 between cells r1c7 and r3c9. The 3rd link is formed with the candidates of digit 6 between the cell r3c9 and the cell r3c6. Finally, the 4th is established between cell r3c6 and r2c4 with the candidate 8.

The two end points r1c2 and r2c4 have the same candidate 1 not linked, so one of the cells must contain this candidate. As such, one may eliminate candidate 1 from the intersection of the final cells (yellow *)

A clear explanation of the formation of the links in a XY chain, named by the author Golden Chain, belongs to Eduyng Castano. [**5**]

2) In the case in which an XY chain is formed out of identical cells which contain the same two candidates X and Y, a Remote Pairs (XY/RP chain) is obtained.

In fig.3, an **XY/RP chain** is formed out of four cells which contain only the candidates 3 and 9. [**6**]

140080060786931254020064801591346728234817596678090413450623180810459602062178045

The XY/RP chain consists of the cells r3c1 (1)-r9c1 (2)-r9c7 (3)-r1c7 (4). If the initial cell (1) r3c1 is 3, then cell (2) r9c1 will be 9, cell (3) r9c7 will be 3 and cell (4) r1c7 will be 9.

If in cell (1) will are 9, in the final cell (4) will be 3.

The house of the initial cell r3c1 is formed from row 3, column 1 and box 1 (light green); the house of the final cell is made up from row 1, column 7 and house 3 (turquoise). The intersections of the final cell's houses are colored yellow. In the end points of the chain we have different values simultaneously and regardless of the placed digits in these cells, **both candidates 3 and 9 are excluded from the intersections r1c3 and r3c8.**

From the intersection r1c3 the candidates 3 and 9 will be excluded; from the intersection r3c8 the candidate 3 will be excluded.

1	4	~~359~~	257	8	25	39	6	79
7	8	6	9	3	1	2	5	4
39	2	359	57	6	4	8	~~37~~	1
5	9	1	3	4	6	7	2	8
2	3	4	8	1	7	5	9	6
6	7	8	25	9	25	4	1	3
4	5	79	6	2	3	1	8	79
8	1	37	4	5	9	6	37	2
39	6	2	1	7	8	39	4	5

Fig.3

Exercise. Configure the XY/RP chains in the grids below and eliminate the cells from the intersection.

a/ 915620408634908102872014096269045081451860000783192645146089003328401060597236814

([**6**] Sudoku solver)

b/ 0038060208003074166000005038738264090219753684500981273000032869380609050900508340

([**7**] Sudoku solver)

c/ 5079206002800607906948173524290705607082069000006090027045739206060002079972681435

([**8**] , fiendish , 12.14.2006)

3) In standard XY chains, as those presented in figures 1 and 2, the repartition of links between the cells is alternating, according to the graphs below:

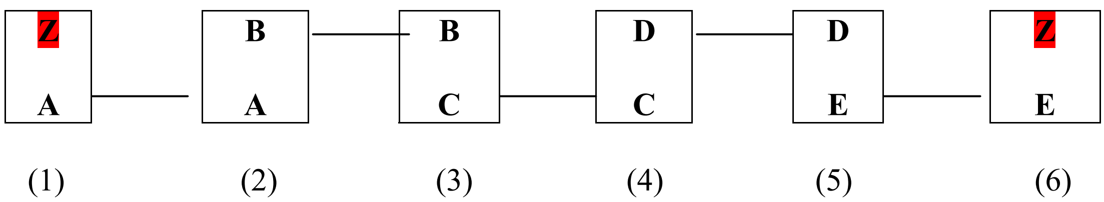

Fig. 4

In Fig. 4, candidates Z are “unlinked” in the final cells (1) and (6); in fig. 5 the candidate 1 is unlinked in the end points (1) and (5).

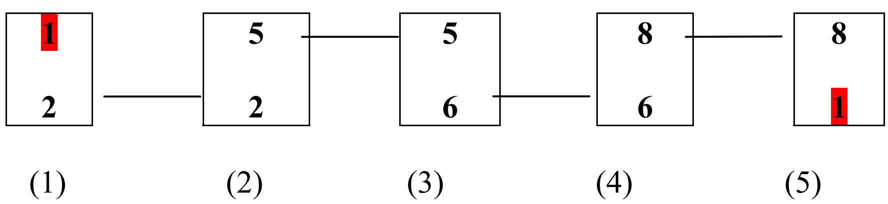

Fig. 5

In an XY/RP chain one can have X-X and Y-Y links and these cannot be separated univocally(?) between the two cells. The bottom row:

AB—AB—AB—AB—AB—AB

(1) (2) (3) (4) (5) (6)

a) considering arbitrarily that in cell 1 we have A "unused" to form new links, then also in cell 6 we have A "unlinked". Therefore **(1)=A => (6)<>A**

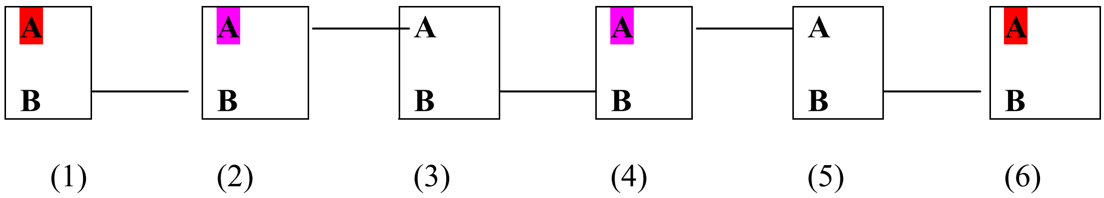

Fig. 6

The chain is valid and candidate A is eliminated from the intersections of cells 1 and 6.

b) considering arbitrarily that in start cell (1) we have B "unused" in a link, B is also unbound in cell (6):

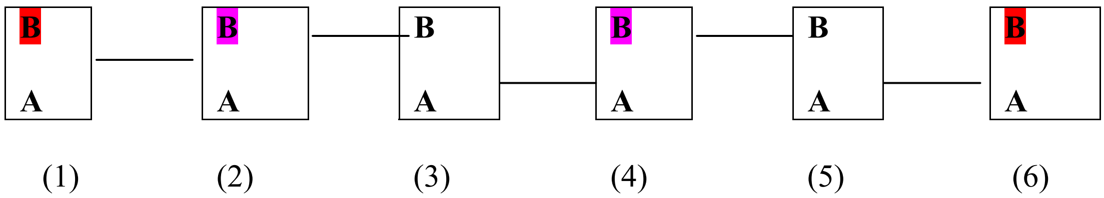

Fig. 7

(1)=B => (6) <>B; and this chain is valid and will determine the elimination of candidate B from the intersection of cells (1) and (6).

The 2 schemes **3a)** and **3b)** are equivalent. By superposing them we come to the conclusion that in a XY/RB chain, made up from identical cells which contain only the candidates A and B, both these candidates are eliminated from the intersection of the end points.

4)a) From figures 6 and 7 one can observe that a XY/RP chain is valid (X and Y candidates cannot coexist simultaneously in the marginal cells) only in the cases in which the number of cells is even.

The necessary condition for the existence of a XY/RP chain is that the N number of cells is even.

The N cells in a chain are connected by (N-1) links.

b) The smallest even number is 2, therefore, just on the line, the simplest XY/RP chain is formed of only 2 cells AB connected through a single link. The "chain" AB-AB is simply a **Naked Pair**. The **NP** model implies the elimination of any of the A, B candidates from the other cells in row, column or box with NP [**9, 10**].

The XY/RP chain can be considered as "a collection" of NP, in accordance with A. Stuart's initial model.

c) The eliminations through naked pairs, an easier technique, are made before the use of the XY/RP chain technique. Therefore, the Sudoku community in general does not visualize the intersection cells of XY/RP chains found in the exclusion zone of 2 **NP** cells (which is somewhat improper, as **NP** is also a **XY/RP chain)**. NP is a sub-set of XY/RP.

The chain XY-XY-XY-XY has architecture as presented below. The elimination of the candidates X, Y is made at the intersection of the houses of the end point cells (1) and (4) of the string.

.	---	.	.	.	.	.	.	.
.	**XY (1)**	.	.	*	.	.	.	.
.	---	.	.	.	.	.	.	.
.	---	.	---	**XY (4)**	---	.	.	.
---	**XY (2)**	---	**XY (3)**	---	---	---	---	---
.	---	.	---	---	---	.	.	.

Fig. 8

The house of cell (1) r2c2 is made up from box 1, row 2 and column 2 (light blue). The house of cell (4) r4c5 is made up from box 5, row 4 and column 5 (orange).

According to the model, in the cells r4c2 and r2c5 (yellow), at the intersection of the houses of the cells (1) and final (4), candidate **X** cannot exist. The chain being symmetric in regard to Y also, it is obvious that those intersections cannot contain candidate **Y**. There exist 3 **NP** between the cells XY from: column 2, row 5 and box 5; candidates X and Y are considered previously eliminated from the cells marked with a dash due to the effect of naked pair. It is considered that the XY/RP method is applied only for the intersection marked with * (r2c5), from which any of the X or Y digits is eliminated.

d) In the current definition, a XY/RP chain is formed from a minimum of 4 cells/ 2 linked NP/.

5) In a XY/RP chain formed of N cells, a number of **sub-chains** can exists from a smaller number of cells: N-2, N-4 etc. For example, in a chain made up from 6 cells, with cells numbered (1) to (6), one can form the sub-chains of 4 cells between the cells: (1)…(4), (2)…(5) and (3)…(6). For each sub-chain specific intersections exist in which X and Y eliminations can be performed. In a XY/RP chain made up from 8 cells, there can exist sub chains of 4 and 6 cells etc.

In the chain from 6 cells in the lower grid with the candidates 2 and 9, elimination is performed in a 4 cell sub-chain:

406007831170843056308006740501470683834600027760380410617030090985060370243700160

Exercise. Find the configuration of a XY/RP chain with 6 AB cells which: a) does not have elimination intersection points; b) forms a maximum number of elimination intersections.

6/ a) XY/RP chain does not apply to a chain with *an odd number of cells*, as the end points (which have an odd number of cells between them) *are not antagonistic*. For an odd number of 3 identical cells exists an elegant proof of this fact [**11**].
Let us analyze the grid:
342980750980070243570234089097350402023419570405027390004792835239040007758163924

	c1	c2	c3
r 4	*168*	**9**	**7**
r 5	*68*	**2**	**3**
r 6	**4**	*16*	**5**
r 7	*16*	*16*	**4**
r 8	**2**	**3**	**9**
r 9	**7**	**5**	**8**

The initial cell is r7c1 and the final cell r2c6. If r7c1 is 1, then r7c2 is 2, and r6c2 is 1; of r7c1 is 6, then r6c2 is 2. All that can be known about the end points is that they will have the same placed digit, but one cannot know its identity. No elimination can be performed in the intersections r4c1 and r5c1.

Opposed to a XY/RP chain, a *normal* XY chain can exist for N=3 cells, ZX-XY-ZY, forming a *XY wing model* in which Z candidate is eliminated.

b) In the XY/RP chains with N odd (greater or equal to 5) the intersections for elimination for **the sub-chains with an even number of cells: N-1, N-3**, etc. In the chain with 5 cells (1)-(2)-(3)-(4)-(5) one can form the sub chains (1)-(2)-(3)-(4) and (2)-(3)-(4)-(5).

348126597615749283729853146071394608003061479496087310932475861064910730107630904
061500078357869124000017065105070096479186253036905701603791502710652039592438617
368057091910603080020189306491576823030892164682314579109738602006901030003065910

7) In accordance to the general convention in chain theory, in an **XY/RP chain the ramifications are neglected** in the calculation of the number of sub-chains. The XY/RP *ramified* chain in fig. 9 cannot lead to eliminations, although it has 4 cells:

AB	**---**	**---**	.	.	.
---	**---**	**---**	.	.	.
---	**---**	**AB**	**AB**	**---**	**---**
.	.	**---**	.	.	.
.	.	**AB**	.	.	.
.	.	**---**	.	.	.

Fig. 9

An example of a grid with a ramified XY/RP chain:

007492106040130007100870340004018765768053014510764830675389421923641578481527693

8) The linking of cells in different XY/RP sub-chains can sometimes lead to the *existence of different elimination routes* for the same intersection. In the grid below, in an XY/RP chain with 7 cells, elimination in the intersection r4c3 can occur via a 4 cell sub-chain or a 6 cell sub chain.

901635070364970150700140369030491706147056903096703541603017090470309615019064237

9) In a grid there can exist more *independent* XY/RP chains, formed from the *same AB digits*:

901635070364970150700140369030491706147056903096703541603017090470309615019064237

or *different pairs of digits* (AB and CD):

964853271035092684802460539289674153640205798057908400406089300028040900590026847

10) XY cycles of identical AB cells cannot generate more eliminations according to the XY/RP model, in regard to those by **NP**, X-wing, Swordfish etc. effects. Indeed, in a cyclic chain is formed from a XY/RP chain through the union of the Z candidates in the marginal cells (fig. 4); candidates possessing the digit Z cannot form antagonistic cells, because they are linked. *Death patterns* which have generated disputes regarding the uniqueness of a Sudoku grid can be considered as resulting from the cyclization of a XY/RP chain.

Part II. Coloring XY/RP chain.

11) For XY/RP cycles with a larger number of cells, 8,9,10 etc. establishing the candidates to be eliminated through the rigorous model from 4) can prove laborious, because it implies the separation of 4, 6 and 8 cell subchains, establishing the house for each marginal cell of the subchains and the analysis of the intersections between them. For example the chain [**6**]:

080560701571283946006010008018400679750698014694001080145000860807106490069840107

12) A comparison between a XY chain and a XY/RP chain is presented below in Table 1:

	Characteristic	**XY chain**	**XY/RP chain**
1	number of cells in the chain	any number	even
2	Elimination	only a X candidate	2 XY candidates
3	Type of cells	Bivalue	Bivalue identical
4	Cells in a house	more	only 2

Table 1

The identical cell in a XY/RP chain induces a behavior similar to an X chain, and the chain of **NP** (which implies the existence of a single pair in a house) is synonymous to a *cluster* in the *coloring* method. [**12**]

13) *Remember Simple Coloring Technique*. When in a row, column or box there exists only 2 candidates for a particular digit, one of these must be true and the other false. Between these candidates a strong link is established and they form a conjugate pair. [**13**] One uses 2 colors which visualizes that those candidates of a certain color can all have, simultaneously, the same true or false value.

From the techniques of Simple Coloring, only the TRAP procedure can be applied to a XY/RP chain, which implies the elimination of a candidate outside of the chain, at the intersection of two differently colored cells. One of the colors will correspond to the true candidate, and the candidates at the intersection of the two will be eliminated. In the grid below a XY/RP chain of 8 cells exists. We apply the COLORING TRAP procedure for each of the candidates with the digits 2 and 7 from the XY/RP chain:

568234100149587060003691485094850010681379524350140800836725941405918630910463058

5	6	8	2	3	4	1	79	79
1	4	9	5	8	7	23	6	23
27	27	3	6	9	1	4	8	5
27	9	4	8	5	26	37	1	367
6	8	1	3	7	9	5	2	4
3	5	27	1	4	26	8	79	679
8	3	6	7	2	5	9	4	1
4	27	5	9	1	8	6	3	27
9	1	27	4	6	3	27	5	8

5	6	8	2	3	4	1	79	79
1	4	9	5	8	7	23	6	23
27	27	3	6	9	1	4	8	5
27	9	4	8	5	26	3~~7~~	1	367
6	8	1	3	7	9	5	2	4
3	5	27	1	4	26	8	79	6~~7~~9
8	3	6	7	2	5	9	4	1
4	27	5	9	1	8	6	3	27
9	1	27	4	6	3	27	5	8

The two grids can be superposed due to the effect of NP: in the red colored cells, for instance, candidate 2 is true and in the blue cells candidate 7 is true.

The cumulated effect of the application of the Coloring Trap method to a XY/RP chain is the elimination of 2 candidates outside the chain having the same digits, 2 and 7 as the cell from the chain at the intersection of two different colors (the digit 7 from the yellow cells r4c7 and r6c9).

5	**6**	**8**	**2**	**3**	**4**	**1**	79	79
1	**4**	**9**	**5**	**8**	**7**	23	**6**	23
27	27	**3**	**6**	**9**	**1**	**4**	**8**	**5**
27	**9**	**4**	**8**	**5**	26	3~~7~~	**1**	367
6	**8**	**1**	**3**	**7**	**9**	**5**	**2**	**4**
3	**5**	27	**1**	**4**	26	**8**	79	6~~7~~9
8	**3**	**6**	**7**	**2**	**5**	**9**	**4**	**1**
4	27	**5**	**9**	**1**	**8**	**6**	**3**	27
9	**1**	27	**4**	**6**	**3**	27	**5**	**8**

The Color XY/RP chain method allows the fast visualization of the cells in which elimination can be performed.

A proof of the application of the Coloring Trap procedure to XY/RP chains is presented in [**14**].

Conclusion. The treatment of the Remote Pairs procedure as a particular case of XY chains allows a more accurate formalization of this technique. The eliminations through XY/RP chain are very accessible using Coloring XY/RP chain.

Exercises.

1) Calculate the maximum number of XY/RP chains in a grid.
2) Calculate the maximum number of subchains in a XY/RP chain with N cells.
3) IntoSudoku solver is exceptionally rich in examples of XY/RP chains (50 grids). From this solver we invite you to enjoy the Sudoku grids below.

Chains with 6 cells

406007831170843056308006740501470683834600027760380410617030090985060370243700160
0009107646190748030740081909000000071703091080061487309192746538836125947457839216
509040720021790450470520009940870215817254693052019874704105902205907040190402567
090010035045630007713540068037100009420090301159320706982451673374900002561273894
080003057375982164000075080850714090010056040400098500530009020692830405048520639
050672014402500607176943258065720143210436005743105062620850000084360020000200006
090010435045630007713540068037100009420090301159320706982451673374900002561273894
506901203309026510124573986743652198862009035951038602000200050210395060005000020
342980750980070243570234089097350402023419570405027390004792835239040007758163924

Chains with 7 cells

348126597615749283729853146071394608003061479496087310932475861064910730107630904
061500078357869124000017065105070096479186253036905701603791502710652039592438617
368057091910603080020189306491576823030892164682314579109738602006901030003065910

Chains with 8 cells

4057109267216090456094251075901007043648972511705000699132546788563714922247900513
345712896602059310910306520451000760723561948896000150534000601109604235260135409
080560701571283946006010008018400679750698014694001080145000860807106490069840107
4057109267216090456094251075901007043648972511705000699132546788563714922247900513
568234100149587060003691485094850010681379524350140800836725941405918630910463058

Chains with 9 cells

2009367109170006300367100903245981678596712437613245890701693206920530711032079563002010581803540205026801340175238062050683018631402050314065826208350100580120063

Chains with 10 cells
158700063390056187670318509081632095006501000035000010507163908019805000863000051

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[4] http://www.decabit.com/Sudoku/Techniques/XYChain.aspx

[5] http://www.coverpop.com/sfiles/Sudoku-GoldenChains.pdf
[6] http://www.intosudoku.com/ (sudoku solver)
[7] http://www.madoverlord.com/projects/sudoku.t (sudoku solver)
[8] http://www.sudokuoftheday.com/pages/s-o-t-d.php
[9] http://www.braingle.com/games/sudoku/np.php
[10]http://www.sudopedia.org/wiki/Naked_Pair
[11] http://www.sudokuonline.us/remote_pairs.php
www.sadmansoftware.com/sudoku/technique15.htm
[12] http://www.sudopedia.org/wiki/Chain
[13] http://www.sudopedia.org/wiki/Simple_Colors
[14] http://www.sudocue.net/guide.php#RemotePair

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