Sudoku BUG: An investigation into the potential of this technique

Origins

I have used (Bi-Value Universal Grave) BUG as a simple and effective technique to end a Sudoku board rapidly for quite a long time. By accident I coded a board which had two apparent BUGs, in different units, of course. In this case the BUG character that resulted was the same for both cells and I wondered whether I could validly treat these two cells as a pincer and make an elimination. This worked and after a large number of attempts in other games I have concluded that this is valid. Furth7er I decided to try chaining from one or other pincer (as usually they were not each the same value) until I had a pincer pair that could eliminate. And from time time I found THREE apparent BUGs and the same deletions were possible, though with three "candidates", this could be trickier.

Only recently I looked again at the description entry for BUG in Andrew Stuart's excellent Solver. This has a standard description of BUG with some examples – but links to a blog which goes a bit further. Here I discover that BUG forms BUG+1, BUG+2 and BUG+3 have been worked on and can resolve the board, though using chaining and forcing techniques. BUG+n is used as a term and I supposed a BUG could be found in theory for all nine occurences of a unit – row, column or box. But the key point I took away is that ONE of the identified BUG characters will be correct. So I have treated these as BUG Candidates (BC) exactly as pincer characters that result from many techniques – with the difference that BCs are often NOT the same value. But otherwise I have considered them as candidate values that can be chained on the board.

I have developed a second difference in that I learnt long ago (from Henk Westhuis' excellent Into Sudoku site and solver) that states one should pick the candidate with three occurrences in the unit (this in reference to a standard BUG+1). I have extended this: to be the Most Commonly Occurring Candidate so the BC can be identified as the one which occurs most often in the unit. I can find no proof of this, but this approach clearly works and is much simpler than considering Deadly Patterns. Though I realise that this is taking the technique away from its origins! I had also assumed that BCs could be found in rows, columns, or boxes, but now dissent from this as far as the techniques I describe below are concerned. I do not know why, but equally I do not really understand why the technique I describe works at all!

There is also an important condition that the board is clean, with no deletions un-done. Probably this means a clean board as in Andrew Stuart's excellent solver after the first six steps, and I have adopted this in the examples that follow.

Development

Of course, as currently understood, the technique may be interesting, but it resolves only quite simple situations, and those at the end of the solution of a game.

However, I appear to have progressed the technique somewhat – and this by utilising two things:
(a) That BCs can be manipulated like any other pincer value; and
(b) by identifying BCs using my "most commonly occurring" principle. I refer to BCs occurring in boxes only.

The simple BC is as below – the first example in Stuart's description:

9	123 (2)	34
6	24	5
8	12	7

He identifies the BC as "2" in the second cell of the first row. And according to my technique the most commonly occurring candidate value in the box is also 2 and it relates to the cell with the most values (in this case three – 1 2 3). Here and in the examples below it is simpler to show just a single box and ignore the rest of the board for the purpose of explanation.

Now, identifying BCs as I have done thus gives rise to some other permutations, and involving more unresolved candidates on a board. My next example is below:

46	45	1
69	8	569 (6)
3	2	7

In this box the most commonly occurring candidate (MCC) is 6 (3 times), and my BC cell is row 2 col 3. Note here there is just one multi-value cell.

The next example is significantly different:

24 (4)	1269	1469
245	1259	3
7	258	258

Here there are six occurrences of “2” in the unresolved cells and there are five multi-value cells that contain it. The BC will therefore be in one of those five cells, but we cannot tell which. However, since there is one bi-value also containing “2”, we can assign a BC value of “4” to that cell (as it cannot be 2). This is a commonly occurring configuration.

Further one can continue the same logic, as below

5	79 (7)	6
3489	89 (8)	139
48	2	17

The MCC is 9 and occurs in 4 cells, or which two are bi-value. We can thus assign TWO BC values: so row 1 col 2 is BC7 and also row 2 col 2 is BC8.

This configuration also occurs frequently and there are, indeed, cases with a third bi-value from which a BC can also be derived.

Subsets in Boxes

One final situation – which follows the same logic and seems to work: and the examples appear to sustain this confidence:

578 (5)	35	19
67	39	169 (9)
58	2	4

Here we have 2 MCCs – 5 and 9 both occur three times. Where the unit contains two non-overlapping subsets (here 578, 35, 58 and 19, 39, 169) it is valid to generate two BCs. In this case row 1 col 1 BC5, and row 2 col 3 BC9.

We can even do this where the subsets overlap, but do not conflict, so that we can generate two MCCs – see example below:

8	1	467
457	34 (3)	2
57 (5)	36	9

The two subsets are 467, 457, 34 (BC3 in row 2 col 2), and 467, 457, 57 (BC5 in row 3 col 1).

I start to think that even if the two subsets indicate the same BC bi-value, then this cel can have TWO BC values.

A commonly found pattern of the same type is as below:

2	345	1
6	345	8
7	9	45 (4 & 5)

Here candidate 3 and 4 appear twice: tentatively I assign TWO BC values to row 3 col 3. Example 8 following contains three Boxes with double-BCs. There is no practical problem resolving the extra BC values, and Example 8 solves! More examples are needed to validate this.

The birth of DoubleBUG

With the exception of these last two examples – for which I have NOT at all fully tested the hypothesis – I have convinced myself, through doing hundreds of games, that it is valid to generate BCs using the above types. And that the set of BCs for the whole board can be treated as a set of Pincer Values. I have developed a few techniques, illustrated in the next section. It is essential, however, that a BC is generated for every unit, and I think one can only use boxes (that are not fully resolved or only contain bi-values). This technique does not seem to work using columns or rows.

Through a growing number of examples I have discovered that the DoubleBUG can take its place as a valid technique. No more than any of the advanced techniques does it provide a solution in all cases, but I am using it after the initial “skirmish” on the board – checking Andrew Stuart’s first six conditions (hidden singles, naked or hidden pairs, triples, quads, pointing pairs and box/line reduction. It is vital that this is done before analysing for DoubleBUG.

At a first analysis – a very swift process – one will not normally find a valid MCC for every box that does not contain solved squares or only contain bi-values. Part of the technique is then to locate chains to make eliminations to each deficient unit so that a valid BC can be generated.

It will be up to each player to decide whether the DoubleBUG analysis is worthwhile. And indeed on many occasions finding a suitable XY-Chain in order that all the units might produce a BC, it has turned out that the board is solved without the need for DoubleBUG. But perhaps it will have been the DoubleBUG analysis that lead to the solving deletion using a chain! Equally, others who decide to experiment with this technique will likely find helpful developments that might extend further the use of the DoubleBUG technique.

At this point the configurations that make finding a MCC appear impossible are the following types:

1. A MCC but the box only contains multi-values, i.e. no bi-value contains the MCC:

6	379	4
5	23789	3789
1	289	289

Candidate 9 appears 5 times, and would normally be the MCC but there is no bi-value. Any of the values could be the BC.

2. A MCC where the only candidates containing the value are each bi-values:

35	13	459
37	1678	89
2	36	45

Here candidate 3 is the MCC (appears 4 times) but each occurrence is in a bi-value, so no BC can be designated.

3. More than one MCC (without clear subsets):

128	5	128
7	389	39
129	6	4

Candidates 1, 2, 8 each appear three times. I see no way to identify a BC.

* * *

With experience I have found it is quite often possible to do a XY-Chain deletion which can give valid MCCs across the board.

It is evident that the fewer BCCs there are, the easier it will be to resolve to a solution. And it also a question of experience as to when to do the DoubleBUG analysis. It must certainly be AFTER the board is cleaned (after stages 1–6 of Andrew Stuart's solver), and personally I systematically look for what are the most straightforward eliminations, before attempting a DoubleBUG analysis – I will do Unique Rectangles, and Hinge for example. And maybe a Y-Wing will stand out.

Also, I note that on occasions we can have a full set of BCs, and still be unable to resolve without further removal of candidates by the known techniques generally used. I make a tentative remark, that the availability of DoubleBUG can evolve ones playing technique after the common techniques most accessible to pencil-and-paper players have been carried out. The Double-BUG technique requires a little familiarity but once mastered it becomes an analytic technique that whilst it must be carefully applied, may be easier than searching for complex chains and other patterns. I do not argue that DoubleBUG, when it can be applied, is necessarily the most efficient technique to resolving the board. Merely that it is an interesting (and rather surprising) additional approach. And one certainly to be used alongside the existing armoury of advanced techniques.

Resolution of the DoubleBUG pincers

With experience I have found two techniques useful.

1. Consolidation:

Candidates are like values and can chain around the board, and the equivalent of a deletion can take place except that the result remains a Candidate and not an absolute value. Two candidates can pincer delete and a new candidate arises – but two candidates are eliminated. The objective is to reduce the number of candidates to ONE, which is then a real value for the board, and may allow the board to be solved.

2. Conforming:

Perhaps the most useful step is to chain one BC to another – eliminating the first in the process. Again, I repeat, the candidate values can chain freely, but they remain candidates until one only remains.

Examples

Full game examples are below, with description of the resolution process. The early ones are quite simple, and more challenging ones follow.

In each case there are two or three screenshots of the board: the first is the original Sudoku; the second the board after the first 6 steps of Andrew Stuart's SudokuWiki Solver, and the third (where present) is the board after additional eliminations have been made as described in the text for each example. Where no additonal eliminations were needed, there is no third screenshot. The source codes under the screenshots can be cut and pasted from the PDF to input to the solvers.

Note that in chaining the BCs around the board more than one pathway may be evident – and the two end points may conflict. This leads one to reject the starting value as being invalid. We believe we know that at least ONE BC identified on a board will be valid, which means that some or even all of the others are invalid. In effect we are solving the game by finding the (or one) valid BC. Nonetheless, in the descriptions of resolution with each example I have not applied this but have adopted a pathway which conforms. There are really many ways to resolve on some boards – all approaches may be valid and in a practical Sudoku game a player is not constrained by the need to annotate every move (as I have to attempt to clarify the procedure) but can quickly eliminate BC values by which ever approach presents itself the most readily.

The DoubleBUG Analysis and the Resolution appear complex as described in the examples following, but once the technique is mastered they are all very simple steps that are swift to effect.

Endnote

A limited amount has been written about BUG in the literature, connected with the solvers or on Sodoku blogs. Looking back, there are clearly players whose remarks and whose examples show the start of a fuller understanding of the possibilities of the technique, though I found no-one appearing to develop the techniques here described. Equally there are a number of blog examples of BUG+2 and BUG+3. The idea that one BC will be correct is clearly established. Using the DoubleBUG analysis, it is interesting to solve some of the examples given in blogs using this DoubleBUG technique. What seemed complex to the blogger becomes simple with this technique. I look forward to many being interested and prepared to explore the possibilities I describe and hopefully to use their Sudoku expertise and experience to extend what I tentatively propose here. And, of course, I await the arrival of a demonstration that DoubleBUG – or a part of it – is invalid!

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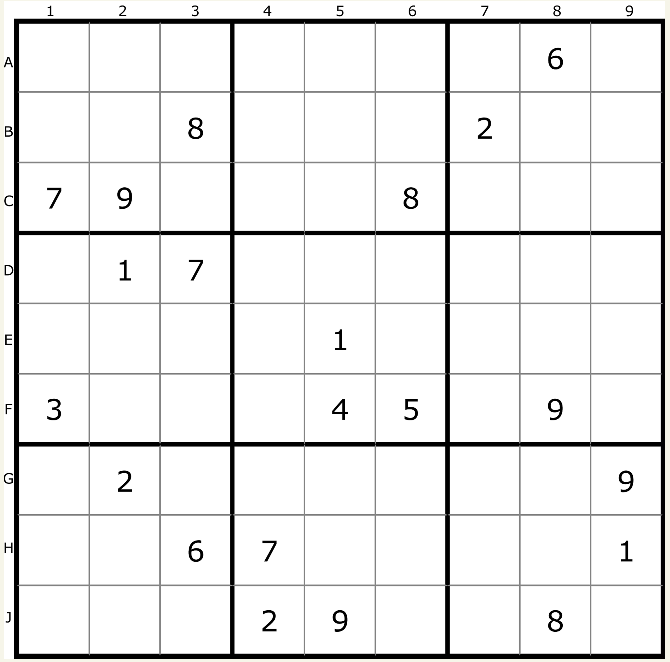
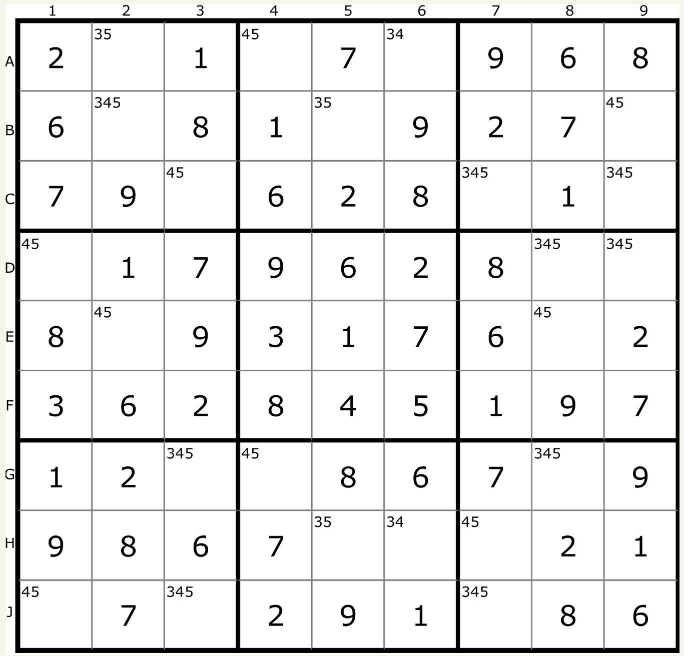
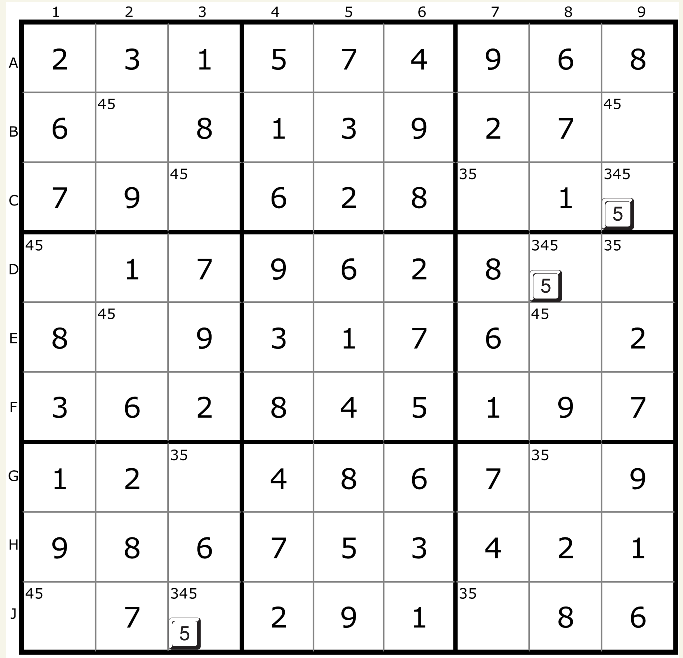
Example 01



Original source:
.......6...8...2..79...8....17..........1....3...45.9..2......9..67....1...29..8.



Source at step 6:
2.1.7.9686.81.927.79.628.1..179628..8.93176.236284519712..867.99867...21.7.291.86



Source at DBUG:
2315749686.813927.79.628.1..179628..8.93176.236284519712.4867.9986753421.7.291.86

This is a simple example, which gives rise to three BCs, and they each happen to be the same value (5). See the right-hand screenshot. After the first 6 steps from the solver (centre screenshot) Boxes 1 and 3 could not produce BCs (see the last example in the preceding text), so two steps were performed:

1. Row B Hinge 4; D9 ≠ 4.

2. XY-CHAIN-4 C3/G4; so J7 ≠ 4.

Both eliminations, together with the accompanying pointing pairs and box/line reductions, reduce the board to the third screenshot. However, my proposal that it is valid to assign TWO BCs to a single cel may change this - see Examples 7 and 8).

DoubleBUG Analysis

The boxes remaining not resolved or with only bi-values are:

Box 3. Three occurrences of 5. DoubleBUG C9$\boxed{5}$

Box 6: Three occurrences of 5. DoubleBUG D8$\boxed{5}$

Box 7: Three occurrences of 5. DoubleBUG J3$\boxed{5}$

Resolution

C9$\boxed{5}$ ... C7$\boxed{3}$ ... J7$\boxed{5}$ ... J1$\boxed{4}$

D8$\boxed{5}$ ... G8$\boxed{3}$ ... G3$\boxed{5}$ ... J1$\boxed{4}$

J3$\boxed{5}$ ... J1$\boxed{4}$

Each of the three BCs resolve by simple chains to J1$\boxed{4}$

This allows the board to resolve fully and simply.

Note: The SudokuWiki Solver requires a simple colouring, and then resolves the game from our DBUG source. From the Source at step 6, the Solver requires one simple colouring, and two XY-Chains.

Example 02

	1	2	3	4	5	6	7	8	9
A	1			5				3	
B	4	2				3			
C		6	3	1	9				
D	9			6	1		5		7
E								2	
F			2			9			
G		8					4		
H				3			6		
J							1	5	3

Original source:
1..5...3.42...3....6319....9..61.5.7.......2...2..9....8....4.....3..6........153

	1	2	3	4	5	6	7	8	9
A	1	79	789	5	2	46	789	3	46
B	4	2	5789	78	678	3	789	16	1569
C	578	6	3	1	9	478	2	48	45
D	9	3	48	6	1	2	5	48	7
E	5678	1457	145678	478	3578	578	389	2	1469
F	5678	1457	2	478	3578	9	38	16	146
G	3	8	156	9	56	156	4	7	2
H	2	1457	1457	3	457	157	6	9	8
J	67	479	4679	2	4678	678	1	5	3

Source at step 6:
1..52..3.42...3....6319.2..93.6125.7.......2...2..9...38.9..4722..3..698...2..153

	1	2	3	4	5	6	7	8	9
A	1	9	78	5	2	4	78	3	6
B	4	2	5	78	6	3	78	1	9
C	78	6	3	1	9	78	2	4	5
D	9	3	4	6	1	2	5	8	7
E	68	17	68	47	3	5	9	2	14
F	5	17	2	478 $\boxed{7}$	78	9	3	6	14
G	3	8	16	9	5	16	4	7	2
H	2	5	17	3	4	17	6	9	8
J	67	4	9	2	78	678 $\boxed{7}$	1	5	3

Source at DBUG:
19.524.36425.63.19.6319.245934612587....3592.5.2..936.38.95.47225.34.698.492..153

This is a simple example, which gives rise to two BCs. See the right-hand screenshot. After the first 6 steps from the solver (centre screenshot) only Boxes 1 and 7 could produce BCs (B2$\boxed{9}$ and J1$\boxed{6}$), so one more step was performed:

1. Col 8 Hinge 8; D3 ≠ 8.

This single step, together with the accompanying pointing pairs and box/line reductions, reduce the board to the third screenshot.

DoubleBUG Analysis

The boxes remaining not resolved or with only bi-values are:

Box 5. Three occurrences of 7. DoubleBUG F4$\boxed{7}$

Box 8: Three occurrences of 7. DoubleBUG J6$\boxed{7}$

Resolution

There are several pathways, for example

F4$\boxed{7}$ ... F5$\boxed{8}$ ... J5$\boxed{7}$

J6$\boxed{7}$

Both J5 and J6 are $\boxed{7}$

Thus J1 = 6, which resolves the board fully and simply.

Note: The SudokuWiki Solver requires a simple colouring, and then resolves the game from our DBUG source. From the Source at step 6, the Solver requires one Y-WING and one X-Cycle.

Example 03

	1	2	3	4	5	6	7	8	9
A	5	4				3		8	
B		3						5	
C			6		9				
D	9							3	1
E			4			7	5		
F				8	1		7		
G						6			
H							3		2
J	6	7			3	5		9	

Original source:
54...3.8..3.....5...6.9....9......31..4..75.....81.7.......6.........3.267..35.9.

	1	2	3	4	5	6	7	8	9
A	5	4	19	16	2	3	69	8	7
B	12	3	129 $\boxed{1}$	167 $\boxed{1}$	78	18	69	5	4
C	7	8	6	5	9	4	2	1	3
D	9	6	7	4	5	2	8	3	1
E	8	1	4	3	6	7	5	2	9
F	23	25	35	8	1	9	7	4	6
G	34	29	38	29	48	6	1	7	5
H	14	59	58	179 $\boxed{1}$	478 $\boxed{8}$	18	3	6	2
J	6	7	12	12	3	5	4	9	8

Source at step 6:
54..23.87.3.....54786594213967452831814367529..819746.....6175......36267..35498

Source at DBUG:

This example gives rise to four BCs in three Boxes. See the centre screenshot: After the first 6 steps from the solver no further steps had to be performed:

DoubleBUG Analysis

The boxes remaining not resolved or with only bi-values are:

Box 1. Three occurrences of 1. DoubleBUG B3$\boxed{1}$

Box 2: Three occurrences of 1. DoubleBUG B4$\boxed{1}$

Box 7: Three occurrences of 1 and 8. Two non-conflicting subsets are identified, each giving rise to a BC:
DoubleBUG H4$\boxed{1}$ and DoubleBUG H5$\boxed{8}$

Note, I eallier was uncertain as to whether it is valid to identify two subsets – however, in this example, and elsewhere the technique appears to work.

Resolution

B3$\boxed{1}$ ... B6$\boxed{8}$ ... H6$\boxed{1}$ ... J4$\boxed{2}$

B4$\boxed{1}$ ... J4$\boxed{2}$

H4$\boxed{1}$ ... J4$\boxed{2}$

H5$\boxed{8}$ ... H6$\boxed{1}$ ... J4$\boxed{2}$

Each of the four BCs resolve by simple chains to J4$\boxed{2}$

Thus J4 = 2 and the board resolves fully and simply.

Note: The SudokuWiki Solver requires an X-Wing and a Y-Wing from step 6.

Example 04

	1	2	3	4	5	6	7	8	9
A				7					
B	4								2
C		6		3					9
D			7	2	8			1	
E		8	1				4		
F						5	3	6	
G			9		6				
H					3		2	8	
J	2	7							

Original source:
...7.....4.......2.6.3....9..728..1..81...4.......536...9.6........3.28.27.......

	1	2	3	4	5	6	7	8	9
A	38	19	25	7	24	69	1568	345	136
B	4	19	38	58	15	69	1678	37	2
C	7	6	25	3	24	18	18 $\boxed{8}$	45	9
D	6	3	7	2	8	4	9	1	5
E	5	8	1	6	9	3	4	2	7
F	9	2	4	1	7	5	3	6	8
G	38	4	9	58	6	2	157	357 $\boxed{3}$	13
H	1	5	6	9	3	7	2	8	4
J	2	7	38	4	15	18	56 $\boxed{6}$	9	36

Source at step 6:
...7.....4.......276.3....963728491558169342792417536 8.49.62. ..15693728427.4...9.

Source at DBUG:

This example gives rise to three BCs in two Boxes – numbers 3 and 9. See the central screenshot. After the first 6 steps from the solver no new steps were performed:

DoubleBUG Analysis

The boxes remaining not resolved or with only bi-values are:

1. Box 3 contains four 1s, and the BC is thus C7$\boxed{8}$.

2. Box 9 contains 3 and 5 three times which make two subsets:

357, 13 and 36 – DoubleBUG G8$\boxed{3}$
157, 357, 56 – DoubleBUG J7$\boxed{6}$

Resolution

C7$\boxed{8}$ ... C6$\boxed{1}$ ... B5$\boxed{5}$ ... B4$\boxed{8}$ ... B3$\boxed{3}$

G8$\boxed{3}$ ... B8$\boxed{7}$ ... A89$\boxed{3}$ ... A1$\boxed{8}$ ... B3$\boxed{3}$

J7$\boxed{6}$ ... J9$\boxed{3}$ ... J3$\boxed{8}$ ... B3$\boxed{3}$

Each of the three BCs resolve by simple chains to B3$\boxed{3}$

B3 = 3, and the board resolves fully and simply.

Note: The SudokuWiki Solver requires two simple colourings, and then resolves the game from the Source at step 6.

Example 05

	1	2	3	4	5	6	7	8	9
A	5						2	4	
B		3	1					8	
C							6		5
D		5	8	7					
E				8					
F	9			6		4			
G			4			9			
H							1	9	7
J	6				2	3			

Original source:
.5.....24..31....8.......6.5.587........8.....9..6.4.....4..9.........1976...23...

	1	2	3	4	5	6	7	8	9
A	5	6	79	39	18	178 $\boxed{1}$	2	4	13
B	4	3	1	2	56	56	7	8	9
C	78	789 $\boxed{7}$	2	39	4	17	6	13	5
D	12	5	8	7	39	12	49	36	46
E	127 $\boxed{1}$	4	6	8	39	125 $\boxed{1}$	59	137 $\boxed{3}$	13 $\boxed{3}$
F	9	17	3	6	15	4	58	127	28
G	18	18	4	5	7	9	3	26	26
H	3	2	5	4	68	68	1	9	7
J	6	79	79	1	2	3	48	5	48

Source at step 6:
56....24.4312..789..2.4.6.5.587......468.....9.36.4.....45793..3254..1976..123.5.

Source at DBUG:

This is a slightly more complex example, but taken directly after the Solver step 6 without further deletions, which gives rise to six BCs in 5 Boxes. See the centre screenshot, No further steps were performed:

DoubleBUG Analysis

The boxes remaining not resolved or with only bi-values are:

Box 1. Three occurrences of 7. DoubleBUG C2$\boxed{7}$

Box 2: Three occurrences of 1. DoubleBUG A6$\boxed{1}$

Box 4: Three occurrences of 1. DoubleBUG E1$\boxed{1}$

Box 5: Three occurrences of 1. DoubleBUG E6$\boxed{1}$

Box 6: Two subsets:
Three occurrences of 1. DoubleBUG E9$\boxed{3}$
Three occurrences of 3. DoubleBUG E8$\boxed{3}$

Resolution

C2$\boxed{7}$ ... C6$\boxed{1}$ ... C8$\boxed{3}$ ... D8$\boxed{6}$ ... G8$\boxed{2}$

E1$\boxed{1}$ and E6$\boxed{1}$ combine to E9$\boxed{3}$

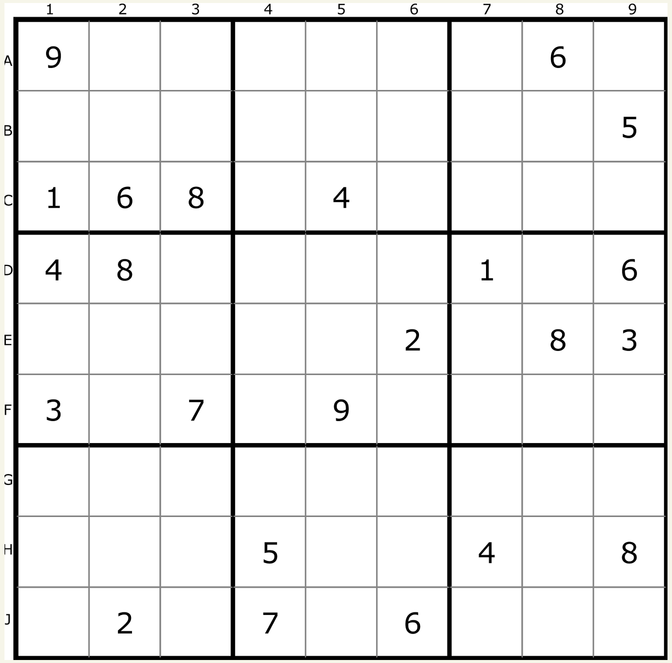
A6$\boxed{1}$ ... A9$\boxed{3}$ pincer with E8$\boxed{3}$ to eliminate E9$\boxed{3}$

E8$\boxed{3}$ ... D8$\boxed{6}$ ... G8$\boxed{2}$

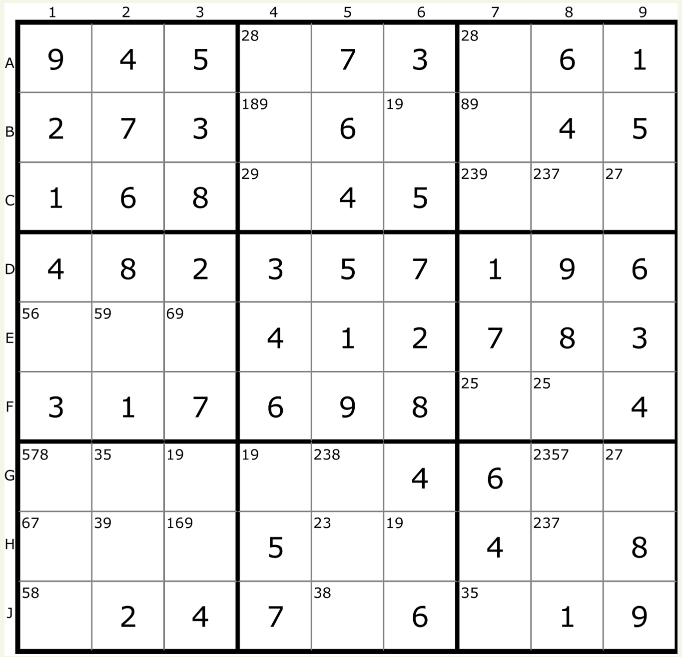
Thus G8 = $\boxed{2}$, and this allows the board to resolve fully and simply.

Note: The SudokuWiki Solver requires a simple colouring, an X-Cycle, and two XY-Chains to resolve the game from the Source at step 6.

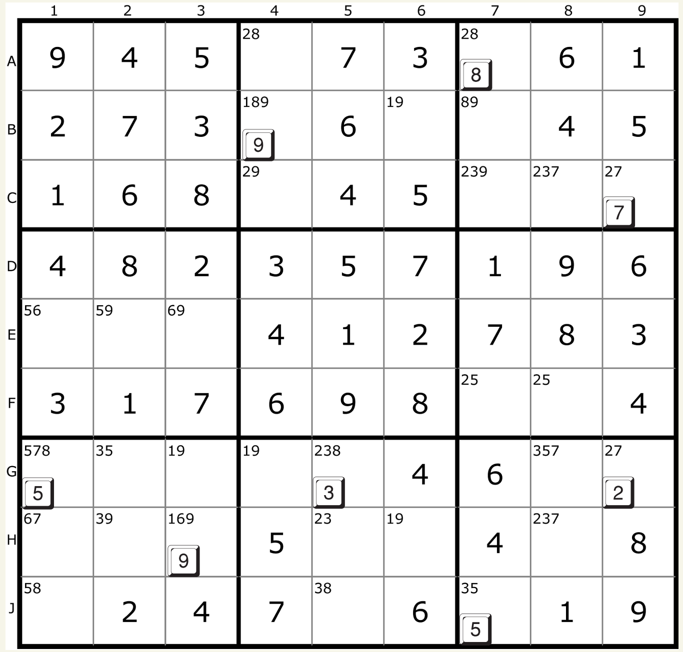
Example 06



Original source:
9......6..........5168.4....48....1.6......2.833.7.9................5..4.8.2.7.6...



Source at step 6:
945.73.61273.6..45168.45...482357196...412783317698..4......46......5..4.8.247.6.19



Source at DBUG:
Same source but G8 ≠ 2

This is a more complex example, which gives rise to eight BCs in 5 boxes. See the right-hand screenshot. After the first 6 steps from the solver (centre screenshot) one step was performed to limit Box 9 to 2 subsets::

1. Col. 9 Unique Rectangle-27: G8 ≠ 2.

This elimination reduces the board to the third screenshot (it has the same source code, and just 1 candidate eliminated).

DoubleBUG Analysis

The boxes remaining not resolved or with only bi-values are:

Box 2. Three occurrences of 9. DoubleBUG B4$\boxed{9}$

Box 3: Three occurrences of 2. DoubleBUG C9$\boxed{7}$

Box 7: Two subsets:
Three occurrences of 5. DoubleBUG G1$\boxed{5}$
Three occurrences of 9. DoubleBUG H3$\boxed{9}$

Box 8: Three occurrences of 3. DoubleBUG G5$\boxed{3}$

Box 9: Two subsets:
Three occurrences of 3. DoubleBUG J7$\boxed{5}$
Three occurrences of 7. DoubleBUG G9$\boxed{2}$

Resolution

A7$\boxed{8}$ ... A4$\boxed{2}$ ... C4$\boxed{9}$... G4$\boxed{1}$ ... H6$\boxed{9}$ ... H2$\boxed{3}$ ... G2$\boxed{5}$ ... J1$\boxed{8}$

B4$\boxed{9}$ ... G4$\boxed{1}$ ... G3$\boxed{9}$ ... H2$\boxed{3}$ ... G2$\boxed{5}$ ... J1$\boxed{8}$

G1$\boxed{5}$ ... J1$\boxed{8}$

H3$\boxed{9}$ ... H2$\boxed{3}$ ... G2$\boxed{5}$ ... J1$\boxed{8}$

G5$\boxed{3}$ ... G2$\boxed{5}$ ... J1$\boxed{8}$

The above five BCs resolve to J1$\boxed{8}$ – which then chains to J7$\boxed{5}$.

The two remaining BCs (C9$\boxed{7}$ and G9$\boxed{2}$) conform to each other but will not chain to J7$\boxed{5}$, so we can say F7 = 5, C9 = 7, G9 = 2, and all other values resolve.

Note: The SudokuWiki Solver requires a Y-WING and an 8-link XY-CHAIN, and then resolves the game from our DBUG source.

Example 07

	1	2	3	4	5	6	7	8	9
A		1		7				6	9
B		9		2				4	
C				6			7		3
D	6	3			4		1		
E			7						
F			5			2	3		4
G	9	4							
H	8							1	
J						3	4		

Original source:
.1.7...69.9.2...4....6..7.363..4.1....7.........5..23.494.......8.......1.......34..

	1	2	3	4	5	6	7	8	9
A	3	1	8	7	5	4	2	6	9
B	7	9	6	2	3	8	5	4	1
C	2	5	4	6	19	19	7	8	3
D	6	3	9	58	4	7	1	25	28
E	4	2	7	3	18	15	689 [8]	59	68
F	1	8	5	9	6	2	3	7	4
G	9	4	12	158	278	56	68 [8]	3	25678
H	8	67	3	4	279	569	69 [9]	1	2567
J	5	67	12	18 [1]	2789	3	4	29	2678

Source at step 6:
3187542697962385412546..783639. 471..4273.....18596237494.....3.8.34...1.5....34..

	1	2	3	4	5	6	7	8	9
A	3	1	8	7	5	4	2	6	9
B	7	9	6	2	3	8	5	4	1
C	2	5	4	6	19	19	7	8	3
D	6	3	9	58	4	7	1	25	28
E	4	2	7	3	18	15	69	59	68
F	1	8	5	9	6	2	3	7	4
G	9	4	12	15	27	56	8	3	2567
H	8	67	3	4	279	569	69 [9]	1	2567
J	5	67	12	18 [1]	2789	3	4	29 [9]	267

Seconf DoubleBUG

This example gives rise to four BCs, in three boxes. See the central screenshot. After the first 6 steps from the solver no further steps were performed.

There is, however, an extra stage to resolution as shown. There should be a better resolution.

DoubleBUG Analysis

The boxes remaining not resolved or with only bi-values are:

Box 6. Three occurrences of 8. DoubleBUG E7[8]

Box 8: Four occurrences of 8. Three are multi-value so the bivalue is the BC. DoubleBUG J4[1]

Box 9. Five occurrences of 6. There are two bi-values giving DoubleBUG G7[8] and H7[9]

Resolution

E7[8] ... E5[1] ... D4[8] ... J4[1]. Eliminate E7[8] which conforms to J4[1]

H7[9] ... E7 ≠ [9] ... E8[9] ... D8[5] ... D4[8] ... J4[1]. Eliminate H7[9] which conforms to J4[1]

J4[1] ... J3[2] ... J8[9]

E7, H7 and J4 conform to J8[9], then ... H7[6] ... G7[8]

Thus G7 = 8. This is a true value, but this does not allow the board to resolve fully. However E7, G459, J9 ≠ 8. Here we review the DoubleBUGs after these deletions. See the right-hand screenshot. There remain only Box 8 (J4[1]) and Box 9 now has H7[9] and J8[2] based on 4 occurrences of 2 and 6. H7[9] continues to resolve to J8[9]. And J4[1] ... J3[2] resolves to J8[9] also. Thus J8 = 9, and the board now resolves fully.

Note: The SudokuWiki Solver required an X-cycle, a WXYZ-Wing, and an XY-Chain to resolve the game from the source at step 6.

Question

This Example contains BCs that will not chain to the other candidates(though the others will chain to it). But in doing the latter, we are prevented from resolving the board. Whilst here I have resorted to the device of a second DoubleBUG analysis, and thus solved the board, this is a situation which has occurred before and in *each* case, the recalcitrant BC is actually the real value for the cel (or in Example 8 both BCs are real values). I conjecture that we can reliably take the BC value as the real one as a standard part of this technique. To resolve ...

Example 08

	1	2	3	4	5	6	7	8	9
A		7					9		6
B	9							5	
C			1						
D		5	8	6					
E			6	9	4		8		1
F		9				3			
G	5	2			8				
H			7			6	3		
J					2		4		

Original source:
..7....9.69......5...1.......586.......694.8.1.9...3...52..8.......7..63.......2.4..

	1	2	3	4	5	6	7	8	9
A	2	7	5	1	3	8	9	4	6
B	9	48	34	2	6	47 $\boxed{4}$ $\boxed{7}$	1	5	378
C	36	468 $\boxed{4}$	1	457	59	479	2	37 $\boxed{3}$ $\boxed{7}$	378
D	4	5	8	6	1	2	7	39	39
E	7	3	6	9	4	5	8	2	1
F	1	9	2	8	7	3	5	6	4
G	5	2	349 $\boxed{3}$	34	8	149 $\boxed{4}$	6	179 $\boxed{9}$	79
H	8	14	7	45	59 $\boxed{5}$	6	3	19	2
J	36	16	39	37	2	179	4	8	5

Source at step 6:
2751389469..26.15...1...2..4586127..736945821192873564 52.. 8.6..8.7..63.2....2.485

Source at DBUG:

This is an advanced example, which gives rise to nine BCs in 6 Boxes. See the centre screenshot. By allowing two BC in a single cel (three times - see text) no further steps were performed.

DoubleBUG Analysis

The boxes remaining not resolved or with only bi-values are:

Box 1. Three occurrences of 4. DoubleBUG C2$\boxed{4}$

Box 2: Two subsets. Three occurrences of 4. DoubleBUG D8$\boxed{7}$; Three occurrences of 7. DoubleBUG B6$\boxed{4}$

Box 3. Two subsets.Three occurrences of 3. DoubleBUG C8$\boxed{7}$; Three occurrences of 7. DoubleBUG C8$\boxed{3}$

Box 7: Three occurrences of 3. DoubleBUG G3$\boxed{3}$

Box 8: Two subsets. Three occurrences of 4. DoubleBUG G6$\boxed{4}$; Three occurrences of 9. DoubleBUG H5$\boxed{5}$

Box 9: Three occurrences of 9. DoubleBUG G8$\boxed{9}$

Resolution

C2$\boxed{4}$ ... H2$\boxed{1}$ ... J2$\boxed{6}$ ... J1$\boxed{3}$ ... J3$\boxed{9}$

B6$\boxed{7}$ will not chain.

B6$\boxed{4}$ ... B3$\boxed{3}$ ... J3$\boxed{9}$

C8$\boxed{7}$ conforms to B6$\boxed{7}$.

C8$\boxed{3}$ ... C1$\boxed{6}$ ... J1$\boxed{3}$ ... J3$\boxed{9}$

G3$\boxed{3}$ ... J3$\boxed{9}$

G6$\boxed{4}$ ... G4$\boxed{3}$ ... G3$\boxed{9}$ ... G9$\boxed{7}$ ... G8$\boxed{7}$ which conforms to B6$\boxed{7}$

H5$\boxed{5}$ ... H4$\boxed{4}$ ... H3$\boxed{1}$ ... J2$\boxed{6}$ ... J1$\boxed{3}$ ... J3$\boxed{9}$

G8$\boxed{9}$ ... G3 $\neq$ $\boxed{9}$ ... J3$\boxed{9}$

Six of the BCs resolve to J3$\boxed{9}$. B6$\boxed{7}$ and C8$\boxed{7}$ which will not chain are taken to be real values: see note in previous example.

So, J3 = 9, B6 = 7, C8 = 7. This allows the board to resolve fully and simply.

Note: The SudokuWiki Solver requires an XY-Chain, and a Y-WING to resolve from the Source at step 6.